IOSR Journal of Applied Geology and Geophysics (IOSR-JAGG)
e-ISSN: 2321–0990, p-ISSN: 2321–0982.Volume 5, Issue 2 Ver. I (Mar. - Apr. 2017), PP 17-22
www.iosrjournals.org

The TPWL Model Order Reduction Method of Accelerating Reservoir Numerical Simulation

CAO Jing, ZHAO Hui, YU Gaoming
（*College of Petroleum Engineering, Yangtze University, Wuhan 430100, China*）

Abstract: *Under the existing calculation conditions, improving reservoir simulation computing speed is of great significance, and it is a hot research issue in the world. At present, although the proper orthogonal decomposition (POD) method can be applied to the nonlinear reservoir simulation system, the acceleration is limited, because in the simulation process, each iteration step requires the construction and projection of the full order Jacobian matrix. Trajectory piecewise-linear (TPWL) reduced order method is be widely used in nonlinear system. The nonlinear system is represented as a weighted combined piecewise linear system. In this paper, the TPWL model reduction method is applied to reservoir simulator, which can greatly reduce the dimension of reservoir model, so as to reduce the calculation time and improve the operation speed.*
Keywords: *reservoir simulation; model order reduction; proper orthogonal decomposition; Trajectory piecewise-linear*

I. Introduction

Reservoir simulation is an indispensable tool. It enables engineers better understand the reservoir physical properties and fluid flow law and to predict hydrocarbon recovery. Traditional reservoir simulators numerically solve a set of governing partial differential equations. This needs solving a set of nonlinear algebraic equations by using iteration. However, as the reservoir simulation models arising from real fields consist of hundreds of thousands or millions of grid blocks, these numerical solutions can be quite time consuming [1]. In addition, if reservoir simulation is used in closed-loop reservoir management [2-5], computational costs are even higher. Where production optimization and the history matching apply repeatedly reservoir simulator, it is extremely time consuming if traditional simulators are used. Therefore, in the case of ensuring the sufficient accuracy of numerical solution, how to greatly accelerate the reservoir simulation speed is the urgent problem to be solved.

Model order reduction (MOR) techniques have shown promise in alleviating computational demands with minimal loss of accuracy [6]. Its task is to reduce the dimension of the state space vector and keep the input and output characteristics of the system at the same time. The proper orthogonal decomposition method (POD) is the most widely used in nonlinear system model reduction method. For now, POD is also widely applied to reservoir simulation. Heijn et al. [7] presented POD method to derive low-order numerical models of two-phase (oil/water) reservoir flow. They illustrated that the POD resulted in a nonlinear model that remained valid over a much longer period, and POD had the potential to improve computational efficiency in the case of multiple simulations of the same reservoir for different well operating strategies. Van Doren et al. [8] applied POD to reduce the dimensions of both the forward model and the adjoint model with the goal of accelerating the optimization of a waterflood process. A 35% reduction in computing time was reported in that work. Cardoso et al. [9] proposed a snapshot clustering and a missing point estimation technique to further accelerate a POD-based reduced-order reservoir simulation model. They achieved speedups of about a factor of 6 to 10.

Although the POD method can be applied to the nonlinear reservoir simulation system, the acceleration is limited, because in the simulation process, each iteration step requires the construction and projection of the full order Jacobian matrix. At present, trajectory piecewise-linear (TPWL) [10] reduced order method is be widely used in nonlinear system. The nonlinear system is represented as a weighted combined piecewise linear system. TPWL method is a non embedded method. It only need to run the reservoir simulator save Jacobian matrix and other derivative matrix in the training process, and build reduced order simulator. You need not to run the full order reservoir simulator in the detection process. In this paper, the TPWL model reduction method is applied to reservoir simulator, which can greatly reduce the dimension of reservoir model, so as to reduce the calculation time and improve the operation speed.

II. Control Equation of Reservoir Model

In this paper, the mathematical model of reservoir model is transformed into the state space equation by means of space discrete in order to explain the reduction process of TPWL method. Two dimensional oil-water two phase reservoir model is used. It is assumed that oil and water do not exchange material, the process is isothermal, the fluid is compressible, and the mass conservation equation and Darcy's law can be used to obtain

[11]:

$$-\nabla \bullet \left[\frac{k_{ri}\rho_i}{\mu_i} \mathbf{K}(\nabla p_i - \rho_i g \nabla d) \right] + \frac{\partial(\phi S_i \rho_i)}{\partial t} - \rho_i q_i^{'''} = 0 \qquad (1)$$

Where $\mathbf{K}$ is permeability tensor; μ is fluid viscosity; k_r is relative permeability; p is pressure; g is gravity acceleration; d is depth; fluid density; ϕ is porosity; S is fluid saturation; t is time; $q^{'''}$ is a source term expressed as flow rate per unit volume; superscript $i \in \{o, w\}$ is respectively oil phase and water phase. In the equation (1), there are four unknown quantities, p_w and S_o are eliminated by using the auxiliary equation (2) and (3), so that only the state variables p_o, S_w are included in the equation,

$$S_o + S_w = 1 \qquad (2)$$

$$p_o - p_w = p_c(S_w) \qquad (3)$$

Where $p_c(S_w)$ is oil-water two-phase capillary pressure.

We consider the relatively simple cases and ignore gravity and capillary force. Format to discrete in space by using five point block centered finite difference, we may have the nonlinear first-order differential equation (4), see the specific derivation of literature [12]:

$$\underbrace{\begin{bmatrix} \mathbf{V}_{wp} & \mathbf{V}_{ws} \\ \mathbf{V}_{op} & \mathbf{V}_{os} \end{bmatrix}}_{\mathbf{V}} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{s}} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{T}_w & \mathbf{0} \\ \mathbf{T}_o & \mathbf{0} \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{F}_w(\mathbf{s}) \\ \mathbf{F}_o(\mathbf{s}) \end{bmatrix}}_{\mathbf{F}} \mathbf{q}_{well,t} \qquad (4)$$

Where: vector $\mathbf{p}$ and $\mathbf{s}$ is grid center oil pressure p_o and water saturation S_w respectively; $\dot{\mathbf{p}}$ and $\dot{\mathbf{s}}$ is the time t derivative of vector $\mathbf{p}$ and $\mathbf{s}$ respectively; $\mathbf{V}$ is the cumulative matrix; $\mathbf{T}$ is transmission matrix; $\mathbf{F}$ is divided flow matrix; Vector $\mathbf{q}_{well,t}$ is the total flow of oil-water well.

Define the state vector $\mathbf{x}$, input vector $\mathbf{u}$ and output vector $\mathbf{y}$

$$\mathbf{x} \triangleq \begin{bmatrix} \mathbf{p} \\ \mathbf{s} \end{bmatrix} \qquad \mathbf{u} \triangleq \begin{bmatrix} \breve{\mathbf{q}}_{well,t} \\ \breve{\mathbf{p}}_{well} \end{bmatrix} \qquad \mathbf{y} \triangleq \begin{bmatrix} \bar{\mathbf{p}}_{well} \\ \bar{\mathbf{q}}_{well,w} \\ \bar{\mathbf{q}}_{well,o} \end{bmatrix} \qquad (5,6,7)$$

Where vector $\breve{\mathbf{q}}_{well,t}$ and $\breve{\mathbf{p}}_{well}$ represent the well of the constant flow and the bottom hole pressure respectively; The vector $\bar{\mathbf{p}}_{well}$ indicates the output bottom hole flow pressure of the constant flow well; Vector $\bar{\mathbf{q}}_{well,o}$ and $\bar{\mathbf{q}}_{well,w}$ indicate the output oil and water flow of the constant bottom hole pressure respectively.

The equation (4) can be written as the form of state space equation [12]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u} \qquad (8)$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x},\mathbf{u}) = \mathbf{C}(\mathbf{x})\mathbf{x} + \qquad (9)$$

In the control system, $\mathbf{A}$ is called the system matrix, $\mathbf{B}$ is called the input matrix, $\mathbf{C}$ is called the output matrix, $\mathbf{D}$ is called the direct transfer matrix. Because the elements of the matrix $\mathbf{V}$, $\mathbf{T}$, $\mathbf{F}$, $\mathbf{J}$ are function of the state variables, the system is a nonlinear system.

III. TPWL Reduced Order Method

By using the TPWL method, a set of linearized points is obtained by using a kind of linear expansion point selection algorithm: $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \cdots, \hat{\mathbf{x}}_{s-1}$. Nnear the linearization points, a set of linear models are obtained by the linear expansion of the nonlinear term $\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x}$:

$$\dot{\mathbf{x}} = \mathbf{G}_i\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i\hat{\mathbf{x}}_i) + \mathbf{B}_i\mathbf{u}, \quad i = 0, 1, \cdots, (s-1) \qquad (10)$$

Where: $\mathbf{G}_i$ is Jacobian matrix of $\mathbf{f}(\mathbf{x})$ at $\hat{\mathbf{x}}_i$, $\mathbf{B}_i = \mathbf{B}(\hat{\mathbf{x}}_i)$.

By using weighted function, the approximate reduction system of the nonlinear system (8) is obtained by weighted summation of the formula (10)

$$\dot{\mathbf{x}} = \sum_{i=0}^{s-1} \omega_i(\mathbf{x})(\mathbf{G}_i\mathbf{x} + (\mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{G}_i\hat{\mathbf{x}}_i) + \mathbf{B}_i\mathbf{u}) \qquad (11)$$

In the literature [10], the proposed algorithm for generating the collection of linearized models may be summarized in the following steps:

1) Generate a linearized model about the initial state $\hat{\mathbf{x}}_0 = \mathbf{x}_0$, and set $i = 0$

2) Simulate the nonlinear system while $\min_{0\le j\le i} \|\mathbf{x} - \mathbf{x}_j\| > \delta$ for some $\delta > 0$,

i.e. while the current state x is close enough to any of the previous linearization points;

3) Generate a new linearized model about $\hat{\mathbf{x}}_{i+1} = \mathbf{x}$, and set $i := i+1$

4) If $i < s-1$, return to step 2.

In the literature [10], the calculation of the weight function $\omega_i(\mathbf{z})$ of the current state $\mathbf{z}$ is as follows:

1) For $i = 0, 1, \cdots, (s-1)$ compute $d_i = \|\mathbf{x} - \hat{\mathbf{x}}_i\|_2$

2) Take $m = \min_{i=0,\cdots,(s-1)} d_i$

3) For $i = 0, 1, \cdots, (s-1)$ compute $\hat{\omega}_{ii} = e^{-\beta d_i/m}$,take $\beta = 25$

4) Normalize $\hat{\omega}_{ii}$ at the evaluation point:

a) compute $S(\mathbf{x}) = \sum_{j=0}^{s-1} \hat{\omega}_j(\mathbf{x})$;

b) For $i = 0, 1, \cdots, (s-1)$, set $\omega_i(\mathbf{z}) = \hat{\omega}_i(\mathbf{z}) / S(\mathbf{z})$.

IV. Example Verification

A numerical example in the literature [12] is used. In this example, a two-dimensional oil-water two phase anisotropic reservoir is described. Its grid is divided into 21 * 21, and the distribution of permeability and porosity is shown in Figure 1, 2. The related parameters of reservoir model: thickness h=2m, length and width of grid $\Delta x = \Delta y$ =33.33m, the viscosity of the crude oil μ_o =5mPa • s, formation water viscosity μ_w =1mPa • s, comprehensive compression coefficient c_t =3.0×10^{-3}MPa^{-1}, the original formation pressure p_i =30MPa, borehole radius r_{well} =0.114m, the end point relative permeability of oil phase k_{ro}^0 =0.9, the end point relative permeability of water phase k_{rw}^0 =0.6, oil phase Corey index n_o =2.0, water phase Corey index n_w =2.0, residual oil saturation S_{or} =0.2, irreducible water saturation S_{wc} =0.2. We use anti five point method well pattern to produce. Center has a water injection well, and four corners have four production wells. We ignore gravity and capillary force.

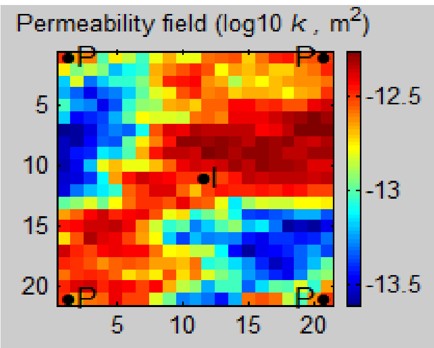

Fig.1 Permeability distribution of reservoir model

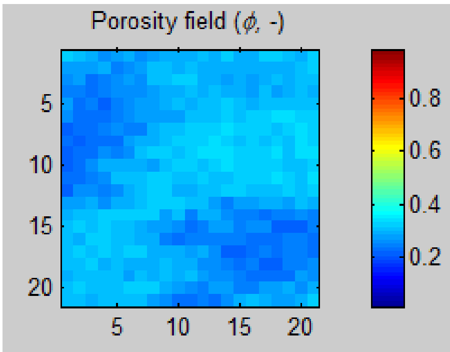

Fig.2 Porosity distribution of reservoir model

The numerical example is simulated by a fully implicit processing. We modify the source code to achieve KPOD model reduction process and verify the validity of the method. It is divided into training and forecasting two processes:

(1) Training process

The bottom hole pressure of production well is 29.5MPa, the bottom hole flow of injection well is $0.0015m^3/s$. We run the full order simulator for 1400 days and save the results of the 66 time steps. The number of selected linearization point is 12.

In the training process, the comparison between the full order reservoir simulator and the reduced order simulator using TPWL method is shown in figure 3, 4.

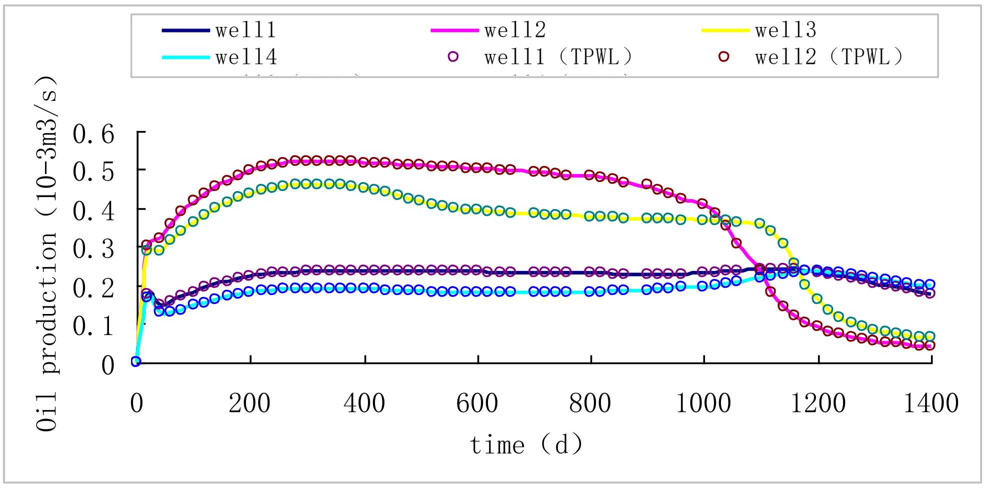

Fig.3 Oil production contrast of four production wells (training process)

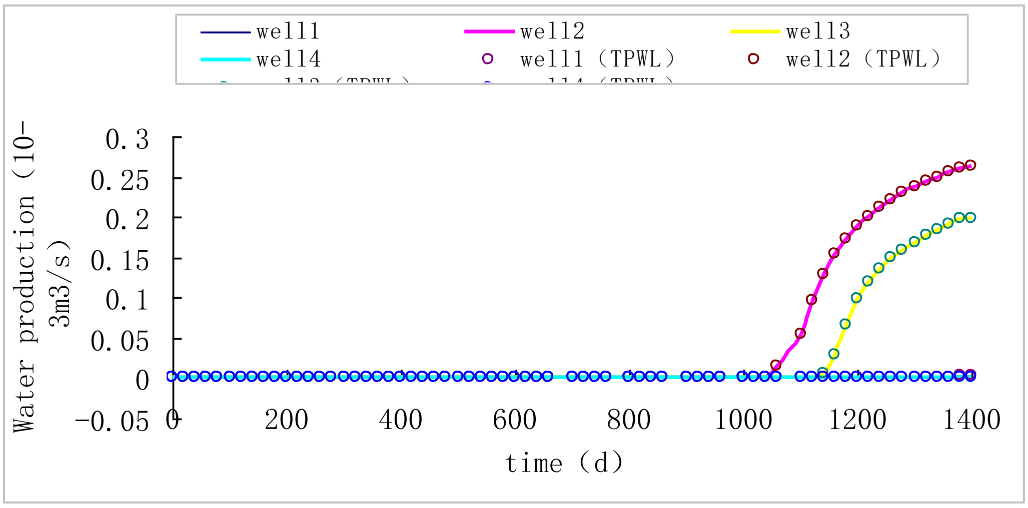

Fig.4 Water production contrast of four production wells (training process)

In this paper, the average relative error is used to measure the accuracy of the approximation. For example, the average relative error of oil production of per well is defined as:

$$\bar{E}_o^m = \frac{1}{n_t}\sum_{i=1}^{n_t}\left|(\mathrm{q}_o^{m,i} - \mathrm{q}_{o,\mathrm{TPWL}}^{m,i}) / \mathrm{q}_o^{m,i}\right|$$

Where i indicates time step; n_t indicates total number of time steps; $\mathrm{q}_o^{m,i}$ indicates the oil production of the full order simulator for the first i step of production well m; $\mathrm{q}_{o,\mathrm{TPWL}}^{m,i}$ indicates the oil production of the reduce order simulator for the first i step of production well m. Similarly, the average relative error $\bar{E}_w^m$ of water production in each production well can be defined.

During the training process, the average relative error of oil production and water yield of four production wells is shown in Table 1.

Table 1 Oil production、water production average relative error of four production wells (training process)

Average relative error %	well1	well2	well3	well4
$\bar{E}_o^m$	0.00874	0.01325	0.01061	0.01264
$\bar{E}_w^m$	0.01542	0.02874	0.0167	0.00001

The above results indicate that in the training process, oil production and water production of four production wells of reduce order and full order simulator are almost identical, the average relative error is very small, but the simulation time is increased nearly 5 times, the running time of the full order simulator is 35.527s, and the running time of reduction simulator is 5.212s.

(2) Forecasting process

At this time, the bottom hole pressure of production wells is changed to 28.5MPa, and the flow rate at the bottom of the injection well remains unchanged. The comparison between the full order simulator and the reduced order simulator is shown in figure 5, 6.

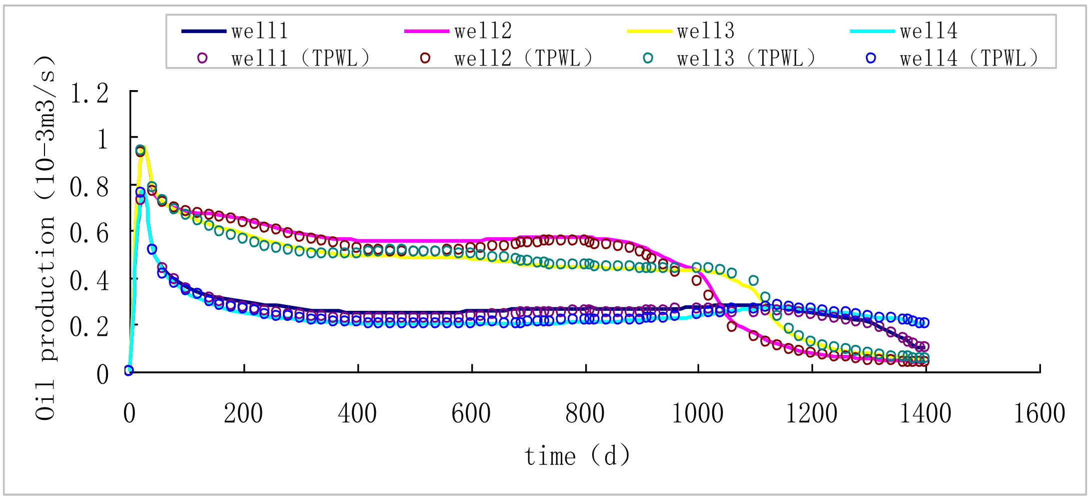

Fig. 5 Oil production contrast of four production wells (prediction process)

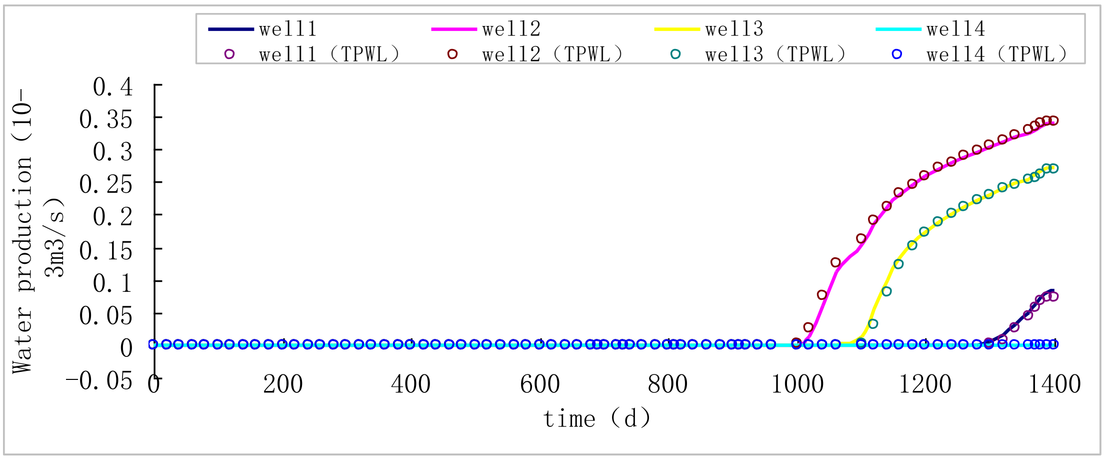

Fig. 6 Water production contrast of four production wells (prediction process)

In the forecasting process, the average relative error of oil production and water yield in four production wells is shown in Table 2.

Table 2 Oil \water production average relative error of four production wells (prediction process)

average relative error %	well1	well2	well3	well4
$\overline{E}_o^m$	4.8932	4.6523	2.879	1.553
$\overline{E}_w^m$	2.642	1.9022	2.1254	0.01

The results show that when the production schedule of forecasting process and training process are different, the average relative error of the reduced order and the full order simulator is improved, but it is still within the reasonable range of 5%. At this time, the simulation time also increases by nearly 5 times. The full order simulator runs for 35.851s, and the running time of the reduced order simulator is 5.411s.

V. Conclusion

1) The application of TPWL model reduced order method to reservoir simulator can greatly reduce the dimension of reservoir model, and improve the operation speed of the simulator by nearly 3 times.
2) When the production schedule of the training and forecasting process is different, the average relative error of the reduced order simulator is improved, but still in a reasonable range of 5%.
3) The improvement of the operation speed of the reservoir simulator provides an important solution for the practical application of the reservoir production optimization and history matching.

Reference

[1]. J. He. Reduced-Order Modeling for Oil-Water and Compositional Systems,with Application to Data Assimilation and Production Optimization. PhD thesis, Stanford University, 2013.

[2]. J. D. Jansen, D. R. Brouwer, G. Navdal, et al. Closed-loop reservoir management. First Break, 2005,23:43−48.

[3]. P. Sarma.Efficient Closed-Loop Optimal Control of Petroleum Reservoirs under Uncertainty.PhD thesis, Stanford University, 2006.

[4]. Chen Y, Oliver D, Zhang D. Efficient ensemble-based closed-loop production optimization [J].SPE J, 2009, 14(4):634-645.

[5]. Zhao Hui, Li Yang, Yao Jun, et al. Theoretical research on reservoir closed-loop production management [J].Sci China Tech Sci, 2011, 54(10):2815-2824.

[6]. David Binion, Xiaolin Chen. A Krylov enhanced proper orthogonal decomposition method for efficient nonlinear model reduction. Finite Elements in Analysis and Design,2011(47): 728–738.

[7]. T. Heijn, R. Markovinovi_c and J. D. Jansen.Generation of low-order reservoir models using system-theoretical concepts. SPE Journal, 2004, 9(2):202-218.

[8]. J. F. M. van Doren, R. Markovinovi ´ c, and J. D. Jansen.Reduced-order optimal control of water flooding using proper orthogonal decomposition. Computational Geosciences,2006, 10:137−158.

[9]. M. A. Cardoso,L. J. Durlofsky,and P. Sarma.Development and application of reduced-order modeling procedures for subsurface flow simulation. International Journal for Numerical Methods in Engineering,2009, 77(9):1322−1350.

[10]. M. J. Rewienski. A Trajectory Piecewise-Linear Approach to Model Order Reduction of Nonlinear Dynamical Systems. PhD thesis, Massachusetts Institute of Technology, 2003.

[11]. Khalid Aziz, Antonin Settari. Petroleum Reservoir Simulation [M]. London: Applied Science Publishers, 1979:128-133.

[12]. J. D. Jansen. Systems Description of Flow Through Porous Media [M].Springer: Springer Briefs in Earth Sciences, 2013:21-36.